Section 8.3: Power Series Solutions to Linear Differential Equations

- Skip section 8.2
- What is an analytic function?
- Ordinary points and singular points
- Other things we need from section 8.2 will be discussed while we do the problems in this section
- Goal: To solve DEs with polynomial coefficients by assuming it's solution is analytic

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Table 1

Important Maclaurin Series and Their Radii of Convergence

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$	$R = 1$
$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} = 1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$	$R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots$	$R = \infty$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots$	$R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots$	$R = 1$
$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{x^n}{n} = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$	$R = 1$
$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \dfrac{k(k-1)}{2!}x^2 + \dfrac{k(k-1)(k-2)}{3!}x^3 + \cdots$	$R = 1$

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4 things you can do to the power series representation of a function
to get a power series representation of another function

1. Replace all x's with another expression
2. Differentiate both sides (might have to shift the index)
3. Integrate both sides
4. Multiply both sides by an expression

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Shifting the index when differentiating both sides of a power series representation of a function

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Example 2 Find a power series solution about $x = 0$ to $y' + 2xy = 0$.

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Example 3 Find a general solution to $2y'' + xy' + y = 0$ in the form of a power series about the ordinary point $x = 0$.

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Example 4 Find the first few terms in a power series expansion about $x = 0$ for a general solution to $(1+x^2)y'' - y' + y = 0$.